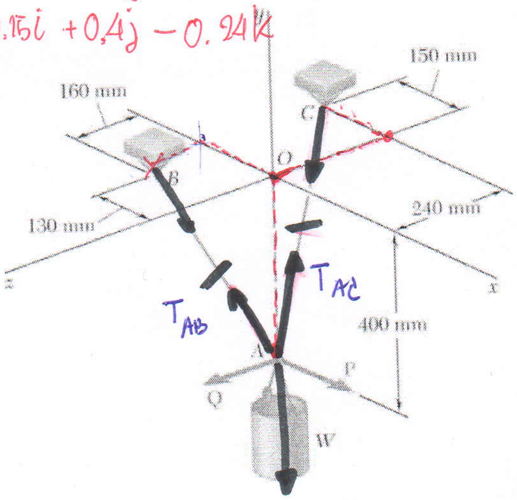
$T_{AB} = T_{AC} = T$

$\vec{r}_{C/A} = -0.15\vec{i} + 0.4\vec{j} - 0.24\vec{k}$



PROBLEM 2.121

A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

$\Sigma\vec{F} = 0 \Rightarrow$ static equilibrium at A

$\vec{P} + \vec{Q} + \vec{T}_{AB} + \vec{T}_{AC} + \vec{W} = 0$

$\vec{W} = -376\vec{j}$

$\vec{P} = P\vec{i}$

$Q = Q\vec{k}$

$i : \Sigma F_x = 0$

$j : \Sigma F_y = 0$

$k : F_z = 0$

$\vec{T}_{AB} = T_{AB}\vec{u}_{AB}$

$\vec{u}_{AB} = \dfrac{\vec{r}_{B/A}}{\vec{r}_{B/A}} \Rightarrow \vec{r}_{B/A} = -0.13\vec{i} + 0.4\vec{j} + 0.16\vec{k}$ *

$|\vec{r}_{B/A}| = \sqrt{0.13^2 + 0.4^2 + 0.16^2}$

$= 0.45$

$$\vec{T}_{AB} = \frac{T \times [-0.13\vec{i} + 0.4\vec{j} + 0.16\vec{k}]}{0.45}$$

$$\boxed{\vec{T}_{AB} = -0.29T\vec{i} + 0.89T\vec{j} + 0.356T\vec{k}}$$

$\vec{T}_{AC} = T_{AC}\vec{u}_{AC}$

$\vec{u}_{AC} = \dfrac{\vec{r}_{C/A}}{|\vec{r}|_{C/A}} \Rightarrow \vec{r}_{C/A} = -0.15\vec{i} + 0.4\vec{j} - 0.24\vec{k}$

$|\vec{r}_{C/A}| = \sqrt{0.15^2 + 0.4^2 + 0.24^2}$

$= 0.49$

$$\vec{T}_{AC} = \frac{T \times [-0.15\vec{i} + 0.4\vec{j} - 0.24\vec{k}]}{0.49}$$

$$\boxed{\vec{T}_{AC} = -0.306\,T\vec{i} + 0.816\,T\vec{j} - 0.49\,T\vec{k}}$$

$$\Sigma\vec{F} = 0$$

$$\vec{P} + \vec{Q} + \vec{W} + \vec{T}_{AB} + \vec{T}_{AC} = 0$$

$$\left.\begin{array}{l} P\vec{i} + Q\vec{k} - 376\vec{j} - 0.29\,T\vec{i} + 0.89\,T\vec{j} + 0.356\,T\vec{k} \\ -0.306\,T\vec{i} + 0.816\,T\vec{j} - 0.49\,T\vec{k} \end{array}\right\} = 0$$

$$\Sigma F_x = 0 \Rightarrow \quad P - 0.29\,T - 0.306\,T = 0$$

$$P - 0.596\,T = 0 \Rightarrow P = 0.596 \times 220.4$$

$$P = 131.36\ \text{N}$$

$$\Sigma F_y = 0 \Rightarrow \quad -376 + 0.89\,T + 0.816\,T = 0$$

$$-376 + 1.706\,T = 0$$

$$T = 220.4\ \text{N}$$

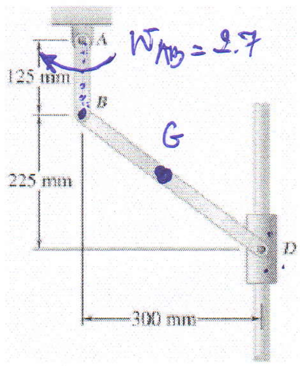
$$\Sigma F_z = 0 \Rightarrow \quad Q + 0.356\,T - 0.49\,T = 0$$

$$Q - 0.134\,T = 0$$

$$Q = 0.134 \times 220.4$$

$$Q = 29.53\ \text{N}$$

$\vec{r} \rightarrow \vec{v} \rightarrow \vec{a} \rightarrow \vec{F} = m\vec{a}$



PROBLEM 15.59

Knowing that at the instant shown the angular velocity of crank *AB* is 2.7 rad/s clockwise, determine (*a*) the angular velocity of link *BD*, (*b*) velocity of collar *D*, (*c*) the velocity of the midpoint of link *BD*.

$\vec{\omega}_{BD}$, $\vec{v}_D$, $\vec{v}_G$

พิจารณา การเคลื่อนที่ ของ Rigid bodies แต่ละชิ้น

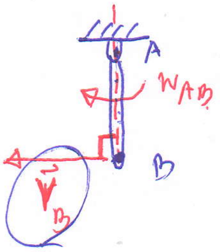
Crank AB ⇒ Rotation about A

Collar D ⇒ Translation

Link BD ⇒ General plane motion

① Crank AB (Rotation)

$-\circlearrowright \quad k \quad i \quad j \quad \circlearrowleft +$

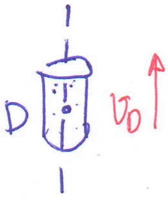


$\vec{v}_B = \vec{W}_{AB} \times \vec{r}_{B/A}$

$= -2.7\vec{k} \times [-0.125\vec{j}]$

$\vec{v}_B = -0.3375\vec{i}$

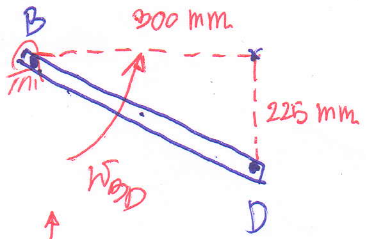
② Collar D. (Translation)



$\vec{v}_D = v_D\vec{j}$

สมมติทิศ ความเร็วของ ของ collar D ↑

③ Link BD (General plane motion)



สมมติทิศ W_{BD} ↺

$\vec{v}_{GP} = \vec{v}_T + \vec{v}_R$

$\vec{v}_D = [\vec{v}_D]_{\text{Translation with } B} + [\vec{v}_D]_{\text{Rotation about } B}$

$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$

$\vec{v}_D = \vec{v}_B + \vec{W}_{BD} \times \vec{r}_{D/B}$

$$v_D \vec{j} = -0.3375\vec{i} + w_{BD}\vec{k} \times [0.3\vec{i} - 0.225\vec{j}]$$

$$v_D \vec{j} = -0.3375\vec{i} + 0.3 w_{BD}\vec{j} + 0.225 w_{BD}\vec{i}$$

เปรียบเทียบสัมประสิทธิ์หน้า i และ j

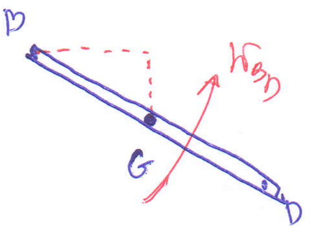
$$i : \quad 0 = -0.3375 + 0.225\, w_{BD}$$

$$w_{BD} = 1.5 \ \text{rad/s} \ \circlearrowleft \quad \text{※}$$

$$j : \quad v_D = 0.3\, w_{BD} = 0.3 \times 1.5$$

$$v_D = 0.45 \ \text{m/s} \ \uparrow \quad \gg$$

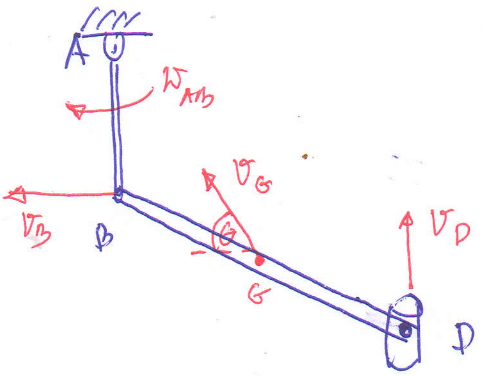
$\vec{v}_G$



$$\vec{v}_G = [\vec{v}_G]_{T \text{ with } B} + [\vec{v}_G]_{R \text{ about } B}$$

$$\vec{v}_G = \vec{v}_B + \vec{v}_{G/B}$$

$$\vec{v}_G = \vec{v}_B + \vec{w}_{BD} \times \vec{r}_{G/B}$$

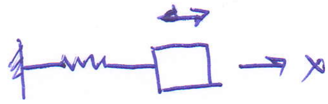


$$\vec{v}_G = -0.1687\vec{i} + 0.225\vec{j} \quad \gg$$

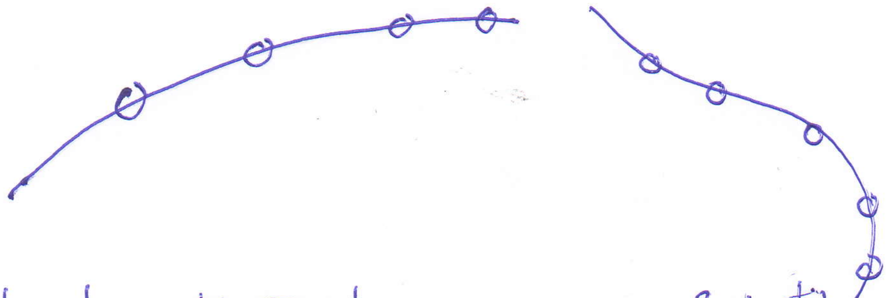
$$\theta = \tan^{-1}\left[\frac{0.225}{0.1687}\right] = 53.13^\circ$$

- การเคลื่อนที่ (Motion)

1. Rectilinear motion



2. Curvilinear motion



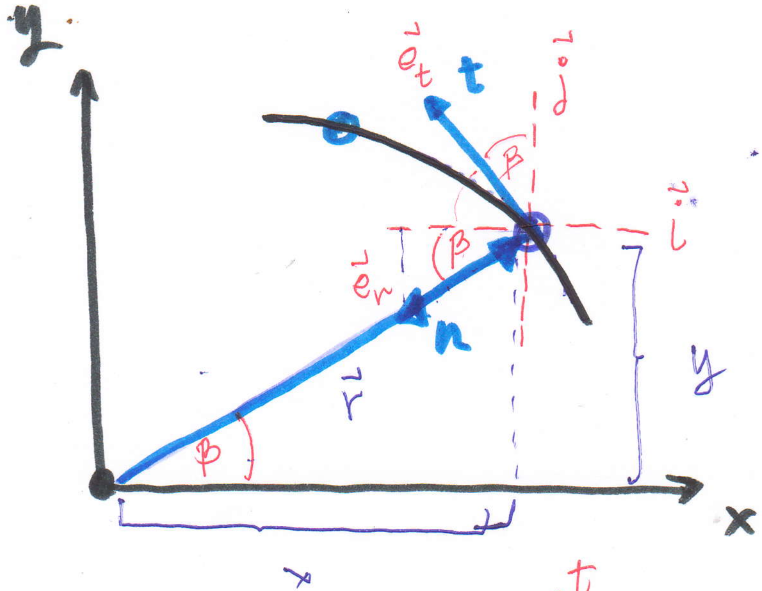
- Rectangular components (x, y) → Projectile < แนว x ⇒ v คงที่ ; แนว y ⇒ a คงที่, a = -g
- Normal & Tangential components (n, t)
- Radius & Transverse components (r, θ)

- Normal & Tangential components (n & t)

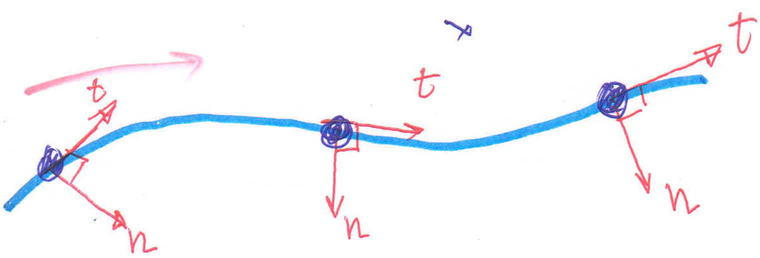
Unit vector ⇒ x ⇒ $\vec{i}$, n ⇒ ? $\vec{e}_n$

y ⇒ $\vec{j}$, t ⇒ ? $\vec{e}_t$

$$|\vec{e}_n| = |\vec{e}_t| = 1$$



$\vec{r} = x\vec{i} + y\vec{j}$



Acceleration : $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d[\rho\dot{\beta}\vec{e}_t]}{dt}$

$$= \rho \frac{d}{dt}[\dot{\beta}\vec{e}_t]$$

$$= \rho \left[\dot{\beta}\frac{d\vec{e}_t}{dt} + \vec{e}_t \frac{d\dot{\beta}}{dt}\right]$$

$$= \rho\left[\dot{\beta}(\dot{\beta}\vec{e}_n) + \vec{e}_t\ddot{\beta}\right]$$

$$= \rho\left[\dot{\beta}^2\vec{e}_n + \ddot{\beta}\vec{e}_t\right]$$

$$\vec{a} = \rho\dot{\beta}^2\vec{e}_n + \rho\ddot{\beta}\vec{e}_t$$

$\dot{\beta} = w$

$\ddot{\beta} = \alpha$ = Angular acceleration

$\rho\dot{\beta}^2 = \rho\left[\frac{v}{\rho}\right]^2 = \frac{\rho v^2}{\rho^2} = \frac{v^2}{\rho}$

$v = wr$

$= \dot{\beta}\rho$

$\dot{\beta} = \frac{v}{\rho}$

$$\vec{a} = \vec{a}_n + \vec{a}_t$$

$\vec{a} = a_x\vec{i} + a_y\vec{j}$

$a_n = \rho\dot{\beta}^2 = \frac{v^2}{\rho}$

$a_t = \rho\ddot{\beta} = r\alpha$

$$\vec{e}_n = -\cos\beta\,\vec{i} - \sin\beta\,\vec{j}$$

$$\vec{e}_t = -\sin\beta\,\vec{i} + \cos\beta\,\vec{j}$$

$$\frac{d\vec{e}_n}{dt} = \sin\beta \times \frac{d\beta}{dt}\,\vec{i} - \cos\beta\,\frac{d\beta}{dt}\,\vec{j} \quad ; \quad \frac{d\beta}{dt} = \dot{\beta}$$

$$= \dot{\beta}\,[\underbrace{\sin\beta\,\vec{i} - \cos\beta\,\vec{j}}_{-\vec{e}_t}]$$

$$\boxed{\frac{d\vec{e}_n}{dt} = \dot{\vec{e}}_n = -\dot{\beta}\,\vec{e}_t}$$

$$\frac{d\vec{e}_t}{dt} = -\cos\beta\,\frac{d\beta}{dt}\,\vec{i} - \sin\beta\,\frac{d\beta}{dt}\,\vec{j}$$

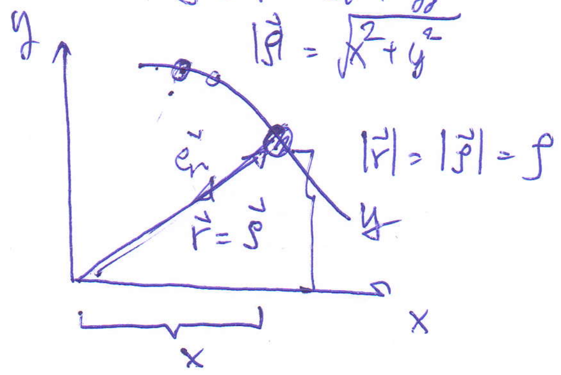
$$= \dot{\beta}\,[\underbrace{-\cos\beta\,\vec{i} - \sin\beta\,\vec{j}}_{\vec{e}_n}]$$

$$\boxed{\frac{d\vec{e}_t}{dt} = \dot{\vec{e}}_t = \dot{\beta}\,\vec{e}_n}$$

$$\vec{V} = v_x\vec{i} + v_y\vec{j}$$

$$\vec{\rho} = \vec{r} = x\vec{i} + y\vec{j}$$

$$|\vec{\rho}| = \sqrt{x^2 + y^2}$$



n e t

Position : $\vec{\rho} = -\rho\vec{e}_n$

Velocition : $\vec{V} = \frac{d\vec{\rho}}{dt} = \frac{d[-\rho\vec{e}_n]}{dt}$

$$= -\left[\rho\frac{d\vec{e}_n}{dt} + \vec{e}_n\frac{d\rho}{dt}\right] \quad ; \quad \rho = \text{คงที่}$$

ขณะที่เราพิจารณา ภาคเคลื่อนที่ ใช้ระบบ n e t

$$= -[\rho(-\dot{\beta}\vec{e}_t) + 0]$$

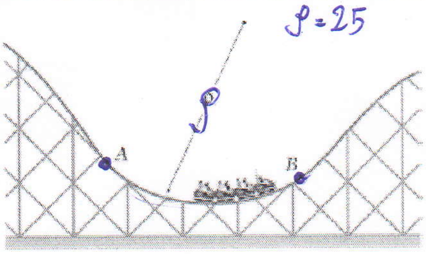
$$\vec{V} = \underline{\rho}\,\dot{\beta}\,\underline{\vec{e}_t} \quad ; \quad \dot{\beta} = \text{Angular velocity} \Rightarrow \omega$$

$$\rho = \text{Radius of curvature} \Rightarrow r$$

$$\vec{V} = \rho\dot{\beta}\vec{e}_t = \underline{r\omega}\,\vec{e}_t$$

$a_n = 3g$ $\frac{d\rho}{dt} = 0$
$= 3\times 9.81$

$\rho = 25$



PROBLEM 11.134

Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion AB of the track if ρ is 25 m and the normal component of their acceleration cannot exceed 3 g.

Answer $(v_{max})_{AB}$ = 97.6 km/h.

$a_n = \frac{v^2}{\rho}$; $v = ?$

$\rho = 25$

$a_n = 3\times 9.81 = 29.43$

$29.43 = \frac{v^2}{25}$

$v = 27.12$ m/s

$v = 27.12 \frac{\cancel{m}}{\cancel{s}} \times \frac{1 \text{ km}}{1000 \cancel{m}} \times \frac{3600 \cancel{s}}{1 \text{ hr}}$

$v = 97.632 \frac{\text{km}}{\text{hr}}$